Crossover from the nonuniversal scaling regime to the universal scaling regime in quantum Hall plateau transitions

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We present in this Brief Report our experimental results on the quantum Hall plateau-to-plateau transition in *long-range Coulombic disordered* two-dimensional electron systems embedded in the $Al_xGa_{1-x}As$-$Al_{0.32}Ga_{0.68}As$ heterostructures (with $x=0\%$ and 0.21%) in a large temperature range from 1.2 K down to 1 mK. In these samples a crossover behavior is observed from the high-temperature, nonuniversal scaling regime to the low-temperature, universal scaling regime, with the temperature exponent κ changing from $\kappa=0.58$ to 0.42, respectively. The crossover temperature increases with increasing x, from 120 mK for $x=0\%$ to 250 mK for $x=0.21\%$. When the Al concentration reaches $x=0.85\%$ at which the short-range random alloy potential dominates the disorder, the crossover temperature is beyond 1.2 K and the universal scaling is observed over two decades of temperature.

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For a two-dimensional electron system (2DES), Abrahams *et al.*[1] argued that in the single-particle picture no extended states can survive in the thermodynamic limit. Thus, at zero temperature (T) all the 2D electron states are localized. What is observed in an experiment done at, inevitably, finite temperatures then depends on the interplay of the localization length ξ, phase coherence length L_ϕ, and sample size L.

In the quantum Hall regime under quantized magnetic (B) field, the perpendicular B field breaks the continuous 2D energy spectrum into Landau subbands, and induces delocalized states at the critical energy E_c (or critical magnetic field B_c) in each subband. The quantum Hall plateau-to-plateau transition, in this description, is a quantum critical phenomenon manifesting the Anderson transition physics of localization-delocalization.[2–4] More specifically, the plateaus indicate that the Fermi level E_F is in the regions of localized states; the transition occurs when E_F is moved through delocalized states at E_c.

The quantum Hall plateau-to-plateau transition is a zero T quantum phase transition. Experimentally, in order to uncover this transition, or any $T=0$ physics, by measurements done at finite T, it is necessary to access temperatures as close to zero as possible and utilize the scaling methodology so that the $T=0$ physics can be reliably extracted and interpreted.[5] In deed, this approach was followed in the first experiment on the integer quantum Hall plateau-to-plateau transition by Wei *et al.*[6] in the late 1980s. They studied the narrowing of the transition region as a function of T and observed that the narrowing, as characterized by the slope of Hall resistance dR_{xy}/dB at B_c, shows power-law dependence, $(dR_{xy}/dB)|_{B_c} \propto T^{-\kappa}$. Here κ is related to the electron localization length ($\xi \propto |B-B_c|^{-\nu}$) critical exponent ν and the electron phase coherence length ($L_\phi \propto T^{-p/2}$) critical exponent p, through $\kappa=p/2\nu$.[7] In their experiments, $\kappa=0.42$, independent of Landau levels involved in the transition.

As an Anderson localization-delocalization transition, it is known that the nature of the disorder in 2DES plays an important role in the quantum Hall plateau-to-plateau transition. In our previous experiments,[8] we have identified three different scaling regimes by tuning the strength of alloy disorder, or the Al concentration, in the $Al_xGa_{1-x}As/Al_{0.32}Ga_{0.68}As$ heterostructure. In the small x regime, $x<0.65\%$, the disorder potential landscape is dominated by the ionized impurities in the modulation doping layer. Thus, the disorder is long ranged and its strength is weak. In this "under-dosed" region, the scaling exponent of the quantum Hall plateau transition is nonuniversal and $\kappa \sim 0.58$ for $x=0\%$ and 0.21%. In the "optimal window" regime of $0.6\% < x < 1.6\%$, short-ranged random alloy fluctuations dominate, the universal scaling behavior of the plateau transition is observed, $\kappa=0.42\pm0.01$. When $x>3\%$, alloy clustering is likely, and scaling exponent becomes nonuniversal again.

Very recently, the investigation of the quantum Hall plateau-to-plateau transition was carried out in an optimal-window sample with $x=0.85\%$ down to a new low-temperature regime, to 1 mK, in a nuclear demagnetization refrigerator.[9] A perfect temperature scaling, $(dR_{xy}/dB)|_{B_c} \propto T^{-0.42}$, was observed through two full decades of tempera-

ture from 1.2 K down to 12 mK. Below $T=10$ mK, a sharp saturation of $(dR_{xy}/dB)|_{B_c}$ occurred. From a systematic sample-size dependences study, this saturation was identified to be a finite-size effect when the quantum phase coherence length reaches *the sample size of millimeters* at ultralow temperatures. This observation allows us to determine the temperature exponent of the inelastic scattering length ($p=2$) in this sample, and a direct measurement of the localization length critical exponent $\nu=2.38$.

The restoration of $\kappa=0.42$ in this optimal-window sample *shows unequivocally that in an Anderson disordered 2DES the universal scaling behavior prevails.* However, questions remain about the physical origin of nonuniversal scaling exponent outside the optimal window, especially for the $x=0$ samples in the under-dosed regime, where a large majority of other scaling experiments were also carried out. It was suspected that the nonuniversality might be related to the long-ranged nature of the disorder and lower experimental temperatures were needed to reach the universal scaling regime.[10,11] Considering that our previous experiments outside of the optimal window were all carried out at relatively high temperature >0.3 K, we here revisit two previously measured under-dosed samples ($x=0\%$ and 0.21%) in a much wider temperature range, down to 1 mk.

In both samples, $\kappa=0.58$ was observed at high temperatures, consistent with our previous results. As the sample temperature was lowered, the universal scaling behavior with the exponent of $\kappa=0.42$ was observed to be restored below a crossover temperature T_0. T_0 increases with increasing Al concentration x, from 120 mK for $x=0\%$, to 250 mK for $x=0.21\%$, and finally to $T_0>1.2$ K in a sample with $x=0.85\%$, where $\kappa=0.42$ was observed from ~10 mK up to 1.2 K.[9] We believe that this crossover behavior in long-range Coulombic disordered systems is due to a transition from thermionic emission at high temperatures to the quantum percolation process at low temperatures, where the quantum phase coherence length exceeds the electron puddle size (L_d) below T_0.

The 2D electron densities for both samples are $1.2\times10^{11}/\mathrm{cm}^2$, and the mobility varies from 3.7×10^6 $\mathrm{cm}^2/\mathrm{V\,s}$ at $x=0\%$ to 2.1×10^6 $\mathrm{cm}^2/\mathrm{V\,s}$ at $x=0.21\%$. In the comparison sample of $x=0.85\%$, the electron mobility is 0.9×10^6 $\mathrm{cm}^2/\mathrm{V\,s}$. In all three samples, the alloy disorder scattering rate can be directly measured by performing temperature-dependence measurements of resistivity at zero B field. For the $x=0\%$ and 0.21% samples, the ratio of alloy scattering rate over total background scattering rate, τ_a^{-1}/τ_b^{-1}, is smaller than 1, while for the $x=0.85\%$ sample, $\tau_a^{-1}/\tau_b^{-1}\sim3.5$.[8]

Magnetotransport measurements at ultralow temperatures were carried out in a nuclear demagnetization refrigerator at the high B/T facilities of NHMFL at University of Florida, Gainesville. The base temperature (T_b) of the demagnetization system is below 1 mK. Specially designed cold contacts were used to enable us to cool 2D electrons. Standard lock-in techniques were used to measure the diagonal resistance R_{xx} and the Hall resistance R_{xy} with a current excitation of 1 nA and frequency of 5.7 Hz. Current self-heating was found to be negligible with the experimental excitation of 1 nA.[9] It is known that the temperature scaling of the plateau-to-plateau transition can be obtained either from R_{xy} by $(dR_{xy}/dB)|_{B_c}\propto T^{-\kappa}$ or from the half width of R_{xx} by $\Delta B\propto T^{\kappa}$. Since the measurement on R_{xx} results in the same exponent, we here concentrate on the Hall resistance R_{xy}. R_{xx} is checked at a few temperatures, and is always consistent with the R_{xy} measurement.

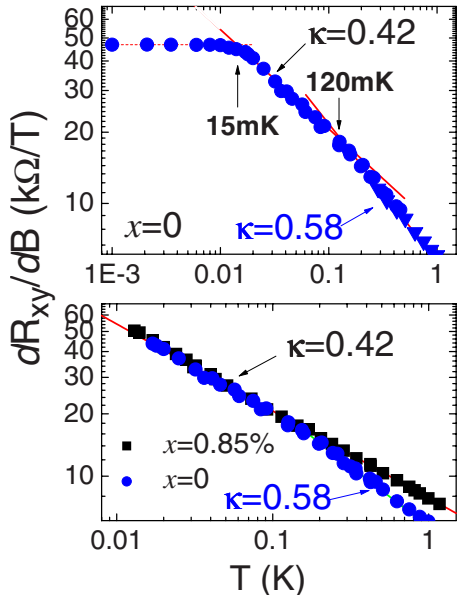

FIG. 1. (Color online) (a) The temperature scaling of $(dR_{xy}/dB)|_{B_c}$. Three different regimes of temperature dependence are observed: $(dR_{xy}/dB)|_{B_c}$ saturates in the lowest temperature decade below 15 mK; power-law dependence with $\kappa=0.58$ in the highest temperature decade; power-law dependence with the universal exponent $\kappa=0.42$ in the middle temperature decade. The crossover temperature between the regions with $\kappa=0.58$ and 0.42 is 120 mK, by extrapolations. (b) Comparison between the $x=0\%$ sample and $x=0.85\%$ sample.

In this work, we focus on the transition from the plateau of filling factor $\nu=4$ to the plateau of filling factor $\nu=3$ (4–3 transition). The critical exponent κ is extracted by fitting the temperature scaling form $(dR_{xy}/dB)|_{B_c}\propto T^{-\kappa}$, shown in Fig. 1(a) for the sample with $x=0$. The sample temperature spans three decades from 1 K down to 1 mK. Three different scaling behaviors are observed in different temperature regions. In the lowest temperature regime of $T<15$ mK, $(dR_{xy}/dB)|_{B_c}$ saturates. This saturation behavior was identified to be a finite-size effect when the quantum phase coherence length L_Φ reaches the sample size at the saturation temperature T_s.[9] To the high-temperature end, the power-law fit to $(dR_{xy}/dB)|_{B_c}\propto T^{-\kappa}$ yields a $\kappa=0.58$. This value of the exponent is identical to that from our previous data taken in a pumped ^{3}He system with the sample temperature varying between 0.3 and 1.2 K. The most striking feature in this plot is the middle temperature range between 15 and 130 mK. Here, the universal critical exponent $\kappa=0.42$ is restored. By extrapolating the linear parts of $\kappa=0.42$ in the middle decade of T and $\kappa=0.58$ in the high decade of T, a crossover temperature $T_0=120$ mK is obtained at the intersection of the extrapolations.

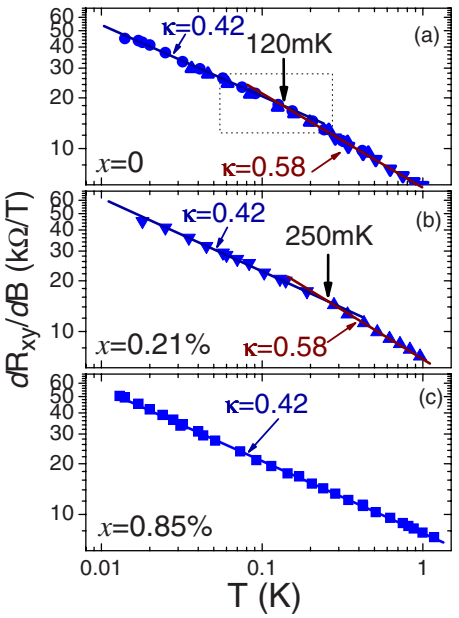

FIG. 2. (Color online) Evolution of the crossover behavior as a function of aluminum concentration x (saturation data in the lowest temperature decade removed) (a) $x=0\%$; (b) $x=0.21\%$; and (c) $x=0.85\%$. The crossover temperature T_0 is marked by arrows.

In Fig. 1(b), we compare the data in Fig. 1(a) (without the saturation part) with that obtained in the $x=0.85\%$ sample from Ref. 9 (also without the saturation part). In the low T region and $T<120$ mK, the two sets of data overlap with each other almost perfectly, with the universal scaling exponent $\kappa=0.42$. For $T>120$ mK, the data in the $x=0\%$ sample clearly deviates from the universal scaling behavior and the exponent becomes $\kappa=0.58$.

We have further investigated this crossover behavior in a sample with $x=0.21\%$. In this sample, a similar crossover behavior is observed, from the high-temperature regime with an exponent $\kappa=0.58$ to the low-temperature regime with the universal scaling exponent $\kappa=0.42$ below $T_0=250$ mK.

Figure 2 shows the "evolution" of T_0 for all three samples. It is clearly seen that T_0 increases with the Al concentration x, from $T_0=120$ mK at $x=0\%$ to $T_0=250$ mK at $x=0.21\%$, and to a temperature beyond our highest measurement temperature of 1.2 K at $x=0.85\%$. We should point out that both the $x=0\%$ and 0.21% samples belong to the so-called regime I in Ref. 8, the under-dosed alloy region, where the alloy scattering rate τ_a^{-1} is less than the total background scattering rate τ_b^{-1}, $\tau_a^{-1}/\tau_b^{-1}<1$. The sample with $x=0.85\%$ belongs to regime II in Ref. 8, the optimal alloy disorder region, where $\tau_a^{-1}/\tau_b^{-1}>1$.

We also want to emphasize here that the large temperature range in our experiments is essential to observe the crossover effect shown in Figs. 1 and 2. Had our measurements been limited in a small temperature range around the crossover temperature, an exponent, different from both 0.42 and 0.58, would have been obtained. Indeed, as shown in Fig. 3, a power-law fitting to the temperature scaling over a relatively small temperature range, from 55 to 280 mK, around the crossover temperature in the $x=0$ sample would result in an exponent $\kappa=0.49$.

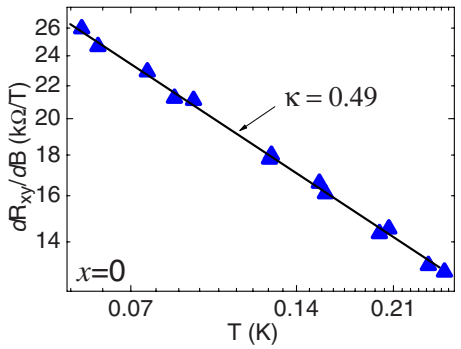

FIG. 3. (Color online) The zoom in of the area marked by the dotted rectangle in Fig. 2(a). A power-law fitting over this relatively small temperature range gives an apparent exponent of $\kappa=0.49$.

In the following, we propose that the crossover observed in the under-dosed samples from a nonuniversal scaling behavior to the universal one is due to the transition from thermionic emission at high temperatures to the quantum percolation process at low temperatures. In the under-dosed regime, $\tau_a^{-1}/\tau_b^{-1}<1$ and disorder is dominantly the weak, long-range Coulombic potential due to ionized modulation doping impurities. As a consequence, electrons form puddles in the 2D channel.[12] Different from the Al alloy impurity dominated case where the physics is more appropriately captured in the Anderson localization picture, here for the weak long-range Coulombic potential, the more appropriate physical picture is that of percolation through saddle points in the potential landscape between puddles.[13–17] Close to the percolation threshold, when the transition between plateaus takes place, charge transport through the sample arises from quantum tunneling through, as well as thermionic emission over, the potential barrier at various saddle points. At high temperatures, the temperature dependence is dominated by thermionic emission over the barriers.[18] Quantum tunneling becomes dominant only at low temperatures when the thermionic emission process is suppressed. Universal scaling can become observable at sufficiently low temperatures, when the electron phase coherence length L_ϕ exceeds typical puddle size L_d, $L_\phi>L_d$, and coherent tunneling prevails. The temperature dependence then is a result from that of L_ϕ. In other words, only in the temperature regime when the quantum percolation process dominates the transport properties at the transition will the temperature scaling exponent κ be directly related to the localization length critical exponent ν in $\xi\sim|B-B_c|^{-\nu}$ through $\nu=p/2\kappa$.

With the above model, we now examine the observation of an increasing T_0 with increasing x. Adding alloy scattering centers into the 2DES channel decreases the relative weight of the background long-range Coulombic potential fluctuations in the disorder landscape, and thus reduces the effective puddle size. The requirement of $L_\phi>L_d$ can now be met at relatively higher temperature. As a consequence, T_0 becomes higher.

In summary, we have observed a crossover behavior from the high-temperature, nonuniversal scaling region with

$\kappa=0.58$ to the low-temperature, universal scaling region with $\kappa=0.42$ in two samples with $x=0\%$ and 0.21%, respectively. The crossover temperature T_0 increases with increasing x or short-ranged alloy disorder. We interpret this crossover behavior as due to a transition from thermionic emission process at high temperatures to a quantum percolation process at low temperatures, where the quantum phase coherence length exceeds the electron puddle size below T_0.

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